Derivatives Test Review

1. $f(x) = \frac{1}{x^3} - 2x^4 + \sqrt{x}$

$f(x) = x^{-3} - 2x^4 + x^{1/2}$

$f'(x) = -3x^{-4} - 8x^3 + \frac{1}{2}x^{-1/2}$

$f'(x) = \frac{-3}{x^4} - 8x^3 + \frac{1}{2x^{1/2}}$

2. $f(x) = 4\sqrt{x} - \frac{3}{\sqrt{x}} + 2x - 5$

$f(x) = 4x^{1/2} - 3x^{-1/2} + 2x - 5$

$f'(x) = 2x^{-1/2} + \frac{3}{2}x^{-3/2} + 2$

$f'(x) = \frac{2}{x^{1/2}} + \frac{3}{2x^{3/2}} + 2$

3. $\frac{d}{dx}(2x^2 \sec x)$

$(4x)\sec x + (2x^2)\sec x \tan x$

4. $\frac{d}{dx}(4x\tan^2(3x^4))$

$\frac{d}{dx}(4x(\tan(3x^4))^2)$

$(4)(\tan(3x^4))^2 + (4x)2(\tan(3x^4))(\sec^2(3x^4))(12x^3)$

$4\tan^2(3x^4) + 96x^4\tan(3x^4)\sec^2(3x^4)$

5. $f(x) = \frac{2x^2}{\sin(3x)}$

$f'(x) = \frac{(4x)\sin(3x) - (2x^2)\cos(3x)(3)}{\sin^2(3x)}$

$f'(x) = \frac{4x\sin(3x) - 6x^2\cos(3x)}{\sin^2(3x)}$

6. $f(x) = 3x^4\sqrt{3x^2 - 5x}$

$f(x) = 3x^4(3x^2 - 5x)^{1/2}$

$f'(x) = (12x^3)(3x^2 - 5x)^{1/2} + (3x^4)\frac{1}{2}(3x^2 - 5x)^{-1/2}(6x - 5)$

$f'(-1) = (12(-1)^3)(3(-1)^2 - 5(-1))^{1/2} + \frac{3(-1)^4(6(-1) - 5)}{2\sqrt{3(-1)^2 - 5(-1)}}$

$f'(-1) = (-12)\sqrt{8} + \frac{3(-11)}{2\sqrt{8}} = -12\sqrt{8} - \frac{33}{2\sqrt{8}}$

7. $f(x) = \ln(5x^2-3)$

$$f'(x) = \frac{10x}{5x^2-3}$$

8. $\frac{d}{dx} 6^{\cos(2x)}$

$$6^{\cos(2x)}(\ln 6)(-\sin(2x))(2)$$

9. $2xy^2+5y-4=8$

$$\left[(2)y^2+(2x)(2y)\frac{dy}{dx}\right]+5\frac{dy}{dx}=0$$

$$2y^2+4xy\frac{dy}{dx}+5\frac{dy}{dx}=0$$

$$4xy\frac{dy}{dx}+5\frac{dy}{dx}=-2y^2$$

$$\frac{dy}{dx}=\frac{-2y^2}{4xy+5}$$

10. $f(x)=\dfrac{5e^{3x}\sqrt{2x-5}}{\sin^2(2x)}$

$$y=\frac{5e^{3x}(2x-5)^{1/2}}{(\sin(2x))^2}$$

$$\ln y = \ln 5 + 3x\cancel{\ln e} + \tfrac{1}{2}\ln(2x-5) - 2\ln(\sin(2x))$$

$$\frac{1}{y}\frac{dy}{dx} = 0 + 3 + \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{2x-5} - 2\cdot\frac{\cos(2x)(2)}{\sin(2x)}$$

$$\frac{1}{y}\frac{dy}{dx} = 3 + \frac{1}{2x-5} - 4\cot(2x)$$

$$\frac{dy}{dx} = \left(3+\frac{1}{2x-5}-4\cot(2x)\right)\left(\frac{5e^{3x}\sqrt{2x-5}}{\sin^2(2x)}\right)$$

11. $f(x)=2x(3x^2-4)^2$ @ $x=1$

$$f(x)=2x(9x^4-24x^2+16)$$

$$f(x)=18x^5-48x^3+32x$$

$$f'(x)=90x^4-144x^3+32$$

$$f'(1)=90(1)^4-144(1)^3+32 = -22$$

12. $g(x) = \ln(2x^3+5)$

$$g'(x) = \frac{6x^2}{2x^3+5}$$

$$g'(2) = \frac{6(2)^2}{2(2)^3+5} = \frac{24}{21} = \frac{8}{7}$$

$h'(2) = {}^{-}7/8$

13. $f(x) = 3e^{2x}$

$f'(x) = 3e^{2x}(2)$

$f'(x) = 6e^{2x}$

14. $\frac{d}{dx}\left(\log_5(\cos(4x))\right)$

$$\frac{-\sin(4x)(4)}{\cos(4x)(\ln 5)}$$

$$\frac{-4\tan(4x)}{\ln 5}$$

15. $f(x) = x^{2x-1}$

$\ln y = \ln x^{2x-1}$

$\ln y = (2x-1)\ln x$

$\frac{1}{y}\frac{dy}{dx} = (2)\ln x + (2x-1)\frac{1}{x}$

$$\frac{dy}{dx} = \left(2\ln x + \frac{2x-1}{x}\right)x^{2x-1}$$

16. $3xy^3 - 4x^2 + 2y = -25$ @ $(3,1)$

$\left[(3)y^3 + (3x)(3y^2)\frac{dy}{dx}\right] - 8x + 2\frac{dy}{dx} = 0$

$3y^3 + 9xy^2\frac{dx}{dx} - 8x + 2\frac{dx}{dx} = 0$

$9xy^2\frac{dy}{dx} + 2\frac{dy}{dx} = 8x - 3y^3$

$$\frac{dy}{dx} = \frac{8x-3y^3}{9xy^2+2}$$

$m_{tan} = \frac{8(3)-3(1)^3}{9(3)(1)^2+2}$

$m_{tan} = \frac{24-3}{27+2} = \frac{21}{29}$

$y - 1 = \frac{21}{29}(x-3)$

17. $y-1=\frac{-29}{21}(x-3)$

18. $f(x)=\csc(x)\cos(x)+3x$

$f(x)=\frac{1}{\sin x}\cdot\frac{\cos x}{1}+3x$

$f(x)=\cot x+3x$

$f'(x)=-\csc^2 x+3$

19. $\frac{d}{dx}[\arcsin(2x^3)]$

$$\frac{6x^2}{\sqrt{1-(2x^3)^2}}=\frac{6x^2}{\sqrt{1-4x^6}}$$

20. $\frac{d}{dx}[2x\arctan(3^x)]$

$$(2)\arctan(3^x)+(2x)\cdot\frac{3^x\ln 3}{1+(3^x)^2}$$

$$2\arctan(3^x)+\frac{(2x)3^x(\ln 3)}{1+3^{2x}}$$

21. $\frac{d}{dx}[\sin(x)^{\tan x}]$

$y=\sin(x)^{\tan x}$

$\ln y=\ln(\sin x)^{\tan x}$

$\ln y=\tan x\ln(\sin x)$

$\frac{1}{y}\frac{dy}{dx}=\sec^2 x\ln(\sin x)+\tan x\cdot\frac{\cos x}{\sin x}$

$\frac{dy}{dx}=\left(\sec^2 x\ln(\sin x)+1\right)\sin(x)^{\tan(x)}$